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An Efficient Derivation of Spatial Green's Function of Rectangular Cavity With CGF-CI Technique

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1 Introduction

The main challenge of time domain methods in analysing structures including rectangular cavities is poor convergence due to resonant nature of cavity. Method of moments (MoM) along with integral equation (IE) analysis can be an efficient and versatile solution due to no suffering from numerical dispersion. A key element in IE solution is relevant Green's function of the structure. But the main fact is the large number of unknowns and slow convergence of iterative solvers in computation of spatial Green's function. Techniques like Poisson summation [1] and Ewald summation [2] have been widely used to accelerate the convergence of Green's function derivation. In this paper combination of characteristic Green's function (CGF) and complex image (CI) technique is used for efficient derivation of spatial Green's function of rectangular cavities. First the desired cavity is approximated by a separable structure. Then CGF method proposed an exact double contour integral representation for separable structure. Before applying CI technique, discrete spectrum contribution part of the integrand is modified by replacing accurate reflection coefficients of surface wave (SW) modes from walls of cavity. The main advantage of the proposed CGF-CI method is its simple and rapid implementation. For validation and and verification of speed and accuracy, comparison with MoM+multilevel fast multipole method (MLFMM) is considered [3].

2 CGF-CI formulation for rectangular cavity and numerical results

A 3-D Green's function of an infinitesmall source surrounded by layered media in all three directions (Fig. 1(a)) is governed by the Helmholtz's equation $[\nabla^2 + k_0^2\varepsilon_r(x,y,z)]G_A(x,y,z;x',y',z') = -\delta(x-x')\delta(y-y')\delta(z-z')$, where $\varepsilon_r(x,y,z)$ represents the variation of the relative dielectric constant. It can be shown that the 3-D Helmholtz's equation can be separated into three 1-D equations if

$$\varepsilon_r(x,y,z) = \varepsilon_x(x) + \varepsilon_y(y) + \varepsilon_z(z), \tag{1}$$

where $\varepsilon_\nu(\nu), \nu = x,y,z$ is the relative dielectric constant of a layered stratified normal to ν direction denoted by $\mathcal{N}_\nu$ [4]. It means that the original structure has been decomposed into three layered media shown in Fig. 1(a). For *separable* structures this decomposition can be done exactly and exact analytical solution for spatial Green's function can be obtained in terms of the related CGFs. The CGF of each $\mathcal{N}_\nu$ layered media denoted by $G_{\nu,\nu}$ is governed by the 1-D Helmholtz's equation $\frac{d^2G_{a,\nu}}{d\nu^2} + (\varepsilon_\nu(\nu)k_0^2 + \lambda_\nu)G_{a,\nu} = -\delta(\nu - \nu')$ where $\nu = x,\ y,\ z$ and $\lambda_x + \lambda_y + \lambda_z = 0$. Solution to 3D Helmholtz's equation under the separability assumption is given by double contour integration as [4]

$$G_A(x,y;x',y') = (\frac{-1}{2\pi j})^2 \oint_{C_x}\oint_{C_z} G_{a,x}(x,x',\lambda_x)G_{a,y}(y,y',-\lambda_x-\lambda_z)G_{a,z}(z,z',\lambda_z)d\lambda_x d\lambda_z \tag{2}$$

The contours C_x and C_z, enclose only the singularities of $G_{a,x}$ and $G_{a,z}$ respectively (including branch cuts, branch points and discreet poles singularities), in counterclockwise sense (Fig. 1(e)). For our desired rectangular cavity of Fig. 1(a), such a separation is not rigorously possible [4]. So we have used an approximate separation of (1). Now if one ignores (1) in exterior wedges resulted by combining $\mathcal{N}_x$, $\mathcal{N}_y$ and $\mathcal{N}_z$ then there will be an infinite number of solutions for ε_{x1}, ε_{x2}, ε_{y1}, ε_{y2}, ε_{z1} and ε_{z2}

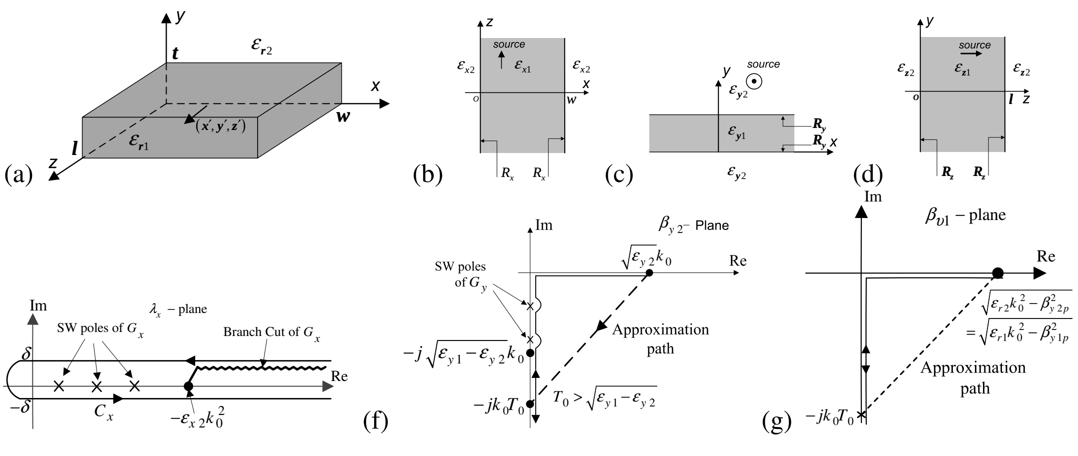

Figure 1. (a) Infinitesmall source in a rectangular cavity, (b) $\mathcal{N}_x$, (c) $\mathcal{N}_y$ (d) and $\mathcal{N}_z$ layered media, (e) Integration path of C_x in λ_x-plane, equivalent and approximate path of C_x in (f) β_{y1}-plane and (g) $\beta_{\nu1}$-plane ($\nu = x,\ z$)

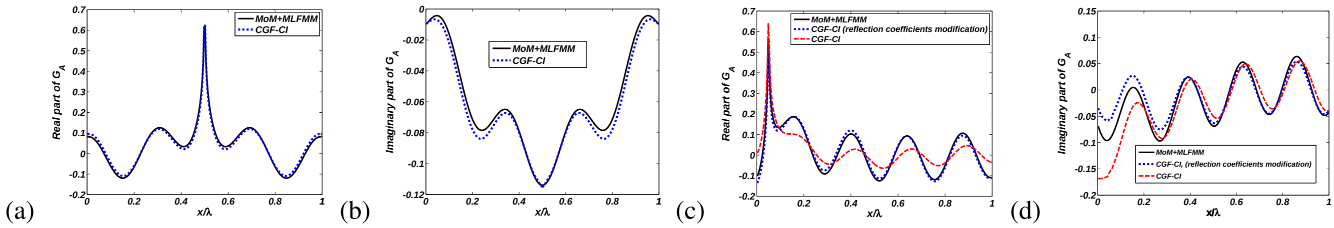

Figure 2. (a) and (b) real and imaginary part of G_A for CGF-CI with the exact MoM+MLFMM for a rectangular cavity with $\varepsilon_{r1} = 20$, $\varepsilon_{r2} = 1$ and $t = 0.1\lambda$, $l = 3\lambda$ and $w = \lambda$, where $y = y' = t$, $x' = w/2$ and $z' = l/2$ and (c), (d) for source located in $x' = w/20$.

satisfy (1). It may be noted that these solutions of $\varepsilon_\nu(\nu), \nu = x, y, z$ are not necessarily physically available relative dielectric constants. Furthermore, it can be shown that the final solution is independent of choice of $\mathscr{N}_x$, $\mathscr{N}_y$ and $\mathscr{N}_z$ layered media [4]. We choose $\varepsilon_{x1} = 0$, $\varepsilon_{x2} = \varepsilon_{r2} - \varepsilon_{r1}$, $\varepsilon_{y1} = \varepsilon_{r1}$, $\varepsilon_{y2} = \varepsilon_{r2}$, $\varepsilon_{z1} = 0$ and $\varepsilon_{z2} = \varepsilon_{r2} - \varepsilon_{r1}$ which help to consider a rectangular cavity as an infinite $\mathscr{N}_y$ dielectric slab which is truncated at $x = 0$, $x = w$, $z = 0$, $z = l$ facets. Having $G_{a,x}$ and $G_{a,y}$ and $G_{a,z}$, numerical computation of (2) is time-consuming due to slowly-convergent and oscillatory behavior of the integrand near the singularities [4]. Combination of CGF and CI has been successfully used for 2-D waveguides [4]. Here in the first step of CGF-CI for double contour integral of (2), surface wave poles of $G_{a,y}$ are extracted. Then the remaining function is accurately approximated as a sum of exponentials along the approximate path indicated in Fig. 1(f). An efficient and fast generalized pencil of function (GPOF) algorithm [5] can be used for this approximation. Then we will have $G_{a,y} \approx \sum_{p=1}^{P_y} \frac{\mathrm{Res}_p}{\lambda_y - \lambda_{yp}} + \sum_{i=1}^{M_y} a_i \frac{e^{b_i \beta_{y2}}}{2j\beta_{y2}}$ where P_y is all the SW poles of $G_{a,y}$ including TE and TM SW poles. Res_p is the residue of $G_{a,y}$ at the pth pole. a_i and b_i come from exponential approximation (GPOF). Substituting in integrand of (2), G_A will be $G_A = G_A^e + G_A^m$. G_A^m represents propagation and incidence of surface wave poles of $\mathscr{N}_y$ medium from an infinite interfaces of $\mathscr{N}_x$ and $\mathscr{N}_z$ media of Fig. 1(b) and Fig. 1(d). But actually the SWs of $\mathscr{N}_y$ are reflected from end-facet of walls of rectangular cavity located in $x = 0$, $x = w$, $z = 0$ and $z = l$ in Fig. 1. This discrepancy results from our separability assumption of original structure of rectangular cavity. In order to improvement of the method, $G_{a,x}$ and $G_{a,z}$ are modified to $G^*_{a,xp}$ and $G^*_{a,zp}$ respectively by replacing the reflection coefficients of pth surface wave pole of $\mathscr{N}_y$ structure at the interfaces of $\mathscr{N}_x$ and $\mathscr{N}_z$ by the correct reflection coefficients from truncating surface like at $x = w$. Now CI technique is applied to $G^*_{a,xp}$ and $G^*_{a,zp}$ to approximate them as a sum of exponentials along the approximate path shown in Fig. 1(g). Then we have $G^*_{a,xp} \approx \sum_{i=1}^{M_{xp}} c_{ip} \frac{e^{d_{ip}\beta_{x1}}}{2j\beta_{x1}}$, $G^*_{a,zp} \approx \sum_{n=1}^{M_{zp}} f_{np} \frac{e^{g_{np}\beta_{z1}}}{2j\beta_{z1}}$ Substituting $G^*_{a,xp}$ and $G^*_{a,zp}$ in (2) and using 2-D Weyl's identity, a closed-form relation of G_A^m can be obtained:

$$G_A^m \approx \frac{-1}{4j} \sum_{p=1}^{P_y} \mathrm{Res}_p \sum_{i=1}^{M_{xp}} \sum_{n=1}^{M_{zp}} c_{ip} f_{np} H_0^{(2)}(k_{\rho p}\rho_{inp}), \tag{3}$$

where $k_{\rho p} = \sqrt{\varepsilon_{r1} k_0^2 - \beta_{y1p}^2} = \sqrt{\varepsilon_{r2} k_0^2 - \beta_{y2p}^2}$ and $\rho_{inp} = \sqrt{(jd_{ip})^2 + (jg_{np})^2}$. It is shown that if the source and also observation points are not very close to corners and walls of cavity, one can assume just the direct source terms of $G_{a,x}$ and $G_{a,z}$ in G_A^e, i.e. $e^{-j2\beta_{\nu 1}|\nu - \nu'|}/(2j\beta_{\nu 1})$, $\nu = x, z$, and then by using the well-known Wely's identity, closed-form relation for G_A^e can be achieved as $G_A^e \approx \sum_{i=1}^{M_y} a_i \frac{e^{jk_0\sqrt{\varepsilon_{r2}}R_i}}{4\pi R_i}$ where $R_i = \sqrt{|x - x'|^2 + (jb_i)^2 + |z - z'|^2}$ and $k_0^2 \varepsilon_{r2} = \beta_{x1}^2 + \beta_{y2}^2 + \beta_{z1}^2$. Then having G_A^m and G_A^e then an approximate closed-form expression for spatial Green's function for rectangular cavity of Fig. 1(a), can be achieved. In Fig. 2, results of real and imaginary parts of G_A for a problem of Fig. 1(a), computed by CGF-CI and MoM+MLFMM are demonstrated for two cases of source locations. In Fig. 2(a) and 2(b) due to sufficient distance of source location from the walls and corners, excellent agreements of CGF-CI results (without any modification of reflection coefficients) with the exact MoM can be achieved. In Fig. 2(c) and (d), source is become closer to left side wall of cavity located in $x' = w/20$, $y' = t$ and $z' = l/2$. We can see that without any reflection coefficients modifications the deviation of the results of CGF-CI in comparison with the exact MoM become larger. With the use of reflection coefficients modification again acceptable agreements can be achieved. The proposed technique is nearly five times faster than MoM+MLFMM.

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